Fluorescence of Dv^{3+} in CaF₂

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The fluorescent "Spectra I" of Dy^{3+} in CaF_2 consisting of approximately 45 lines are found to originate from the crystal-field split levels of the ${}^{4}F_{11/2}$ and ${}^{6}H_{15/2}$, ${}^{6}H_{13/2}$ and ${}^{6}H_{11/2}$ free-ion states, in accord with energy-level schemes which indicate the crystal-field symmetry for Spectra I about the Dy ion to be type Oh group. These schemes also give information on the crystal-field energy level representations and energies to 3 cm⁻¹, preliminarily.

RIVALENT rare-earth-doped fluorite crystal fluorescent line spectra have been observed to fall into two distinct types, "I and II," according to the method of crystal preparation.¹ The theoretical bases for "Spectra I and II" are not completely understood.

A preliminary analysis² of Spectra I and II of Sm³⁺ in CaF_2 has indicated that in this case the fluorescent lines can be accounted for largely as transitions between the crystal-field split levels of the upper fluorescent and lower ground multiplet Russell-Saunders states of the Sm³⁺ free-ion. Based on a frequency shift between corresponding subgroups of Spectra I and II these two spectra were identified² as originating from two different upper fluorescent states of Sm³⁺.

However, there are several other trivalent rare-earthdoped fluorite spectra for which the approximate spectral location of the subgroups are the same¹ for types I and II; and the analysis of the Spectra I and II of one such crystal such as Dy^{3+} in CaF_2 could be helpful in this regard. The purpose of this paper is to indicate the crystal-field symmetry and crystal-field energy level origins, preliminarily, of the Dy3+ in CaF2 Spectra I.³ The basis for this analysis will be several spectrograms obtained in conjunction with a recent investigation⁴; these spectrograms (shown in Fig. 1) record the fluorescent Spectrum I of a crystal of 0.1% Dy3+ in CaF₂ with 0.1% NaF, at liquid-nitrogen temperature, to an accuracy of 5 cm^{-1} .

Since, in rare-earths the crystal field is much smaller than the spin-orbit interaction, each Russell-Saunders state splits independently into crystal-field levels. Group theory may be easily used to find the amount of this splitting and the allowed transitions between such split levels if the group theoretical symmetry about the rareearth ion is known.⁵

In general, it is theoretically possible, in principle, to solve this spectral problem completely, using perturbation theory, by obtaining the wave functions and

energies of the crystal-field split states. However, the amount of labor is prohibitive here and some information required to obtain such exact results may be lacking for a many-electron atom in crystal fields such



WAVE LENGTH - MILLIMICRONS (APPROXIMATE)

FIG. 1. Fluroescence of Dy^{3+} in CaF_2 , Spectrum I. The line numbers are identified in Table I. (A) Enlargement of the spectro-gram of Subgroup A. (B) Enlargement of the spectrogram of Subgroup B. (C) Enlargement of the spectrogram of Subgroup C. (D) Mercury comparison spectrogram. (E) Spectrograms showing the relative positions and intensities of Subgroups A, B, and C.

¹I. V. Stepanov and P. P. Feofilov, Doklady Akad. Nauk. SSSR 4, 615 (1957) [translation: Soviet Phys.-Dokl. 1, 350 (1956)]. ² N. Rabbiner, Phys. Rev. **130**, 502 (1963).

^a The general crystal-field symmetry type of Dy^{3+} in CaF_2 , Spectra II will be seen likely to be of a lower symmetry than that for Spectra I.

⁴H. H. Theissing, P. Caplan, T. Ewanizky, and G. deLhery, Appl. Opt. **2**, 291 (1963). ⁵H. Bethe, Ann Physik **3**, 133 (1929).

Line λ (Å) \bar{v} (cm ⁻¹) I-A I-A B1 4895 20 429 WBB2 4884 20 475 WW3 4864 20 559 BB4 4799 20 838 5 4792 20 868 6 4787 20 990 7 4780 20 921 W8 4774 20 947 9 4769 20 969 10 4767 20 978 11 4764 20 991 W12 4758 21 017 B13 47503 21 053-40 W14 4746 21 070 W15 4735 21 119 W16 4722 21 177 WW17 4717 21 200 I-B I 146 23 5742 17 461 24 5732 17 461 25 5717 17 486 27 5717 17 486 <		by m car ₂ , opeend	
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$	29	5706	17 525
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	30	5703	17 535
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40 0002 15 147 15 164	43	6505	15 147
45 6500 15 104	44 15	6500	15 104
$\frac{1}{46}$ $\frac{1}{6581}$ $\frac{15173}{15105}$	43	6581	15 175
47 6574 15 193	40	6574	15 195
S48 6560 15 273	548	6569	15 223
	D 10	0007	10 440

TABLE I. Frequencies of fluorescent lines of Dy³⁺ in CaF₂, Spectrum I.^a

* See Appendix

as in the trivalent rare-earth doped fluorites. In such cases group theory can provide vital information with comparatively little effort and is a powerful and elegant tool.

It is known that the CaF₂ lattice is octahedral without any rare-earth doping. If the Dy³⁺ ion is then incorporated, lattice distortion ensues. But if the distortions introduced by placing of Dy³⁺ into the CaF₂ lattice are comparatively small, the crystal field will remain O_{h} . A recent investigation⁶ has indicated this to be the case. In this paper the assumption of O_h symmetry for Dy³⁺ in CaF₂ is made; and the deduced crystal-field energy level schemes are found to be in accord with the 45 observed lines to within the accuracy of 5 cm⁻¹. This tends to confirm the O_h crystal symetry, and, in addition, provides information on the crystal-field level energies and representations.

Referring to Fig. 1, the fluorescent spectra of a crystal of Dy^{3+} in CaF_2 is seen to be composed of three subgroups, A, B, and C, and resembles¹ Spectrum I. These spectrograms came from a diffraction grating a linear dispersion averaging 19.5 Å/mm. The frequencies of these lines are listed in Table I under the subgroups A, B, and C to an accuracy of 5 cm⁻¹.

Using Table I, the mean frequencies of these A, B, and C subgroups are found to be 20 900, 17 400, 15 100 cm⁻¹, thus identifying them as the sets of transitions between the free-ion states of known energies (E): the ${}^{6}F_{11/2}$ fluorescent state and the ${}^{6}H_{15/2}$, ${}^{6}H_{13/2}$, and ${}^{6}H_{11/2}$ ground-multiplet states of Dy³⁺. This is in accord with the energies of these states which have been previously established⁷ as follows:

 ${}^{6}F_{11/2}$; $E \sim 20\ 900\ \text{cm}^{-1}$: (A, B, C subgroup origins)

 ${}^{6}H_{11/2}$; $E \sim 5800 \text{ cm}^{-1}$: (C subgroup terminus)

 ${}^{6}H_{13/2}$; $E \sim 3500 \text{ cm}^{-1}$: (B subgroup terminus)

 ${}^{6}H_{15/2}$; $E \sim 000 \text{ cm}^{-1}$: (A subgroup terminus).

TABLE II. Crystal-field splittings and allowed transitions in O_h field^a [$\Gamma_6 \leftrightarrow \Gamma_7$ transition forbidden].

State	Crystal-field split representations	Transitions to ${}^6F_{11/2}$ state		Total number allowed transitions to ${}^6F_{11/2}$ state		
		${}^{6}H_{15/2}$	${}^{6}F_{11/2}$	(Observe	d) De	v.
⁶ H _{15/2}	Γ ₆ , Γ ₇ , Γ ₈ , Γ ₈ , Γ ₈	$\Gamma_6 \Gamma_7$	$\Gamma_6 \Gamma_7$	20 2 - 18	(17)	1
		$\Gamma_8 \ \Gamma_8 \ \Gamma_8$	${\Gamma_8\over \Gamma_8}$	20-2=18 (17) -		
⁶ H _{13/2}	Γ6, Γ7, Γ7, Γ8, Γ8	${}^{6}H_{13/2}$ Γ_{6} Γ_{7} Γ_{7} Γ_{8} Γ_{8}	${}^{6}F_{11/2}$ ${\Gamma}_{6}$ ${\Gamma}_{7}$ ${\Gamma}_{8}$ ${\Gamma}_{8}$	20-3=17	(16)	-1
⁶ H _{11/2}	Г6, Г7, Г8, Г8	${}^{6}H_{11/2}$ ${}^{\Gamma_{6}}_{\Gamma_{7}}$ ${}^{\Gamma_{8}}_{\Gamma_{8}}$ ${}^{\Gamma_{8}}_{\Gamma_{8}}$	${}^{6}F_{11/2}$ Γ_{6} Γ_{7} Γ_{8} Γ_{8}	16 - 2 = 14	(15)	+1

^a See Ref. 5.

⁶ W. Low, Proc. Phys. Soc. (London) 76, 307 (1960).

⁷ G. H. Dieke and S. Singh, J. Opt. Soc. Am. 46, 495 (1956); and G. H. Dieke, H. M. Crosswhite, and B. Dunn, *ibid.* 51, 826 (1961).



FIG. 2. O_h -crystal-field energy-level scheme in accordance with fluorescence frequencies observed for Spectrum I of Dy³⁺ in CaF₂.

The approximate numbers of the lines listed in Table I are those expected for transitions between levels of these free-ion states split (cf Table II) by a crystal field of O_h symmetry using the group-theory splittings and selection rules for J=11/2, 13/2, 15/2 free-ion states. The maximum theoretically allowed number of transitions (cf Table II) according to group theory for subgroups A, B, and C (using the above J-value assignments) are 18, 17, and 14 lines, respectively, compared with the observed numbers of 17, 16, and 15 lines (Table I.)

Using the data listed in Table II for O_h symmetry, energy level schemes (Fig. 2) are deduced which correspond to the observed frequencies to within 3 cm⁻¹ (Table III). These schemes (Fig. 2) seem quite unique for the observed frequencies and may be used to give the level energies and group theory representations⁵ of the crystal-field levels. The above appears to be strong evidence in support of O_h symmetry for Spectra I of Dy³⁺ in CaF₂. This energy level scheme type of check may be helpful in finding or corroborating the crystalfield symmetry in other such crystals.

The general characteristics of Spectra II of Dy^{3+} in CaF_2 which show¹ many more lines than Spectrum I in the same spectral locations, tend to support the notion that the crystal-field symmetry of Spectra II is of a lower symmetry than O_{h_2} allowing greater crystal field splittings of the same Dy^{3+} free-ion states given⁷ above, with a consequently greater number of theoretically allowable transitions.^{2,5}

Note added in proof. Since the writing of this paper, it has come to the attention of the author that investigations by R. W. Bierig and M. J. Weber of the EPR spectrum of Dy^{3+} in CaF₂ at liquid helium temperature

have indicated Γ_8 and not Γ_7 (as reported previously in Ref. 6 by Low) to be the ground state of the cubic-field type crystals. In addition, other EPR resonances due to ions of Dy³⁺ in CaF₂ sites of tetragonal and trigonal local crystal-field symmetries have been reported by these authors. As indicated in this paper, such crystals of lower symmetry spectra are probably associated with type II crystals of Dy³⁺ in CaF₂.

Bierig and Weber have also suggested that the crystal field of Sm^{3+} in CaF_2 may be trigonal or tetragonal (instead of the C_2 and C_3 symmetries proposed previously in Refs. 1 and 2).

In Ref. 2 it had been indicated that Spectrum I originated from four crystal-field-split levels of the second lowest fluorescent state of Sm³⁺ labeled as a $J=\frac{7}{2}$ state, effectively. Now, according to Fig. 2 of Ref. 2, the two states adjacent to $J = \frac{7}{2}$ (labeled $J = \frac{5}{2}$ and $\frac{9}{2}$) have energy separations from the $J=\frac{7}{2}$ state comparable with the crystal-field-split energy splittings of the $J=\frac{7}{2}$ state itself. In such cases where the crystal-field splittings are comparable with separations between free-ion states, J is no longer a good quantum number due to mixing of free-ion states (here the mixing of $J=\frac{5}{2}$ and $\frac{9}{2}$ states with the $J=\frac{7}{2}$ state). The simple group-theoretical analysis based on the splitting an unmixed J state should then be modified for a precise solution; in the above Dy³⁺ case where the crystal-field splittings are small this simple group theory method is precise.

ACKNOWLEDGMENT

I am grateful for the support of the authors of Ref. 4, and in particular to T. Ewanizky for the spectrograms.

$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Line	Exp.	Scheme	Dev.
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1	20 429	20 426	-3
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2	$20\ 475$	$20\ 474$	-1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	3	20 559	20 561	2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	4	20 838	20840	2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	5	$20\ 868$	$20\ 867$	-1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	6	20 890	20 888	-2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	7	20 921	20 919	-2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	8	20 947	20946	-1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	9	20 969	20 967	-2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	10	20978	20975	-3
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	11	20 991	20 988	-3
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	12	21 017	21 015	-2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	13	21 053-40	$21\ 054,\ 41,$	$\{1, 0,$
14 21 070 21 068 -2 15 21 119 21 123 4 16 21 177 21 176 -1 17 21 200 -1 0 18 17 161 17 161 0 19 17 185 17 188 3 20 17 212 17 209 -3 21 17 298 17 296 -2 22 17 388 17 389 1 23 17 416 17 444 -2 25 17 461 17 459 -2 26 17 486 17 486 0 27 17 507 0 0 28 17 507 17 507 0 29 17 525 17 524 -1 30 17 535 17 524 -1 33 17 627 17 625 -2 34 14 883 14 883 0 35 14 910 14 931 -1 36 15 029 0 0 38 15 029 15 029			36	(-4
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	14	21 070	21 068	-2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	15	21 119	21 123	4
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	16	21 177	21 176	-1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	17	21 200	17 4 64	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	18	17 161	17 161	0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	19	17 185	17 188	3
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	20	17 212	17 209	-3
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	21	17 298	17 290	-2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	22	17 388	17 389	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	23	17 410	17 410, 17	0, 1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	24	17 440	17 444	2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	23	17 401	17 439	-2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	20	17 400	17 400	0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	28	17 492	17 490	-2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	20	17 525	17 524	1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	30	17 525	17 538	3
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	31	17 550	17 552	2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	32	17 593	17 504	1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	33	17 627	17 625	-2^{1}
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	34	14 883	14 883	อื
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	35	14 910	14 910	ŏ
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	36	14 932	14 931	-1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	37	14 943	11,01	-
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	38	15 020	15 019	-1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	39	15 029	15 029	Ō
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	40	15 056	15 056	0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	41	15 090	15 089	-1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	42	15 135	15 137	2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	43	15 147	15 147	0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	44	15 164	15 163	-1
46 15 195 15 195 0 47 15 211 1 1 48 15 223 15 224 1	45	15 175	15 174	-1
47 15 211 48 15 223 15 224 1	46	15 195	15 195	0
48 15 223 15 224 1	47	15 211		
	48	15 223	$15\ 224$	1

TABLE III. Deviation of frequencies of energy-level schemes from frequencies observed (cm⁻¹).

APPENDIX: NOTES TO TABLE I

Referring to Fig. 1, the lines of Subgroup B are the strongest and those of Subgroup C, the weakest

(on the average). The strength of a line relative to others in the same subgroup is indicated by the letters: S-stronger, W-weaker, and WW-much weaker. Broad lines are denoted by letters: B-broad and BB-very broad; such broad lines have an observed accuracy which is less than that of the other lines, which are all sharp.

In Subgroup A, line No. 3 at 4864 Å is indicated in Fig. 1(A); it was obscured by the light background. In Subgroup B, Fig. 1(B) shows a very weak mercury line (M_1) at 5790 Å [Fig. 1(D)].

In Subgroup C, Fig. 1(C) shows weak mercury lines (M_2) at 6716 Å and a second-order mercury line 3341.5 Å at 6683 Å (M_3) . These were also indicated in spectrograms not shown here. In the region between line Nos. 41 and 42, there appears to be two very weak lines (labeled 'V') at 6616 Å and 6620 Å. Line No. 37 [6692 Å] which is unaccounted for in Fig. 2, may have originated from lines of the exciting ultraviolet source [which had a filter transparent to the long-wavelength part of the spectrograms at wavelengths >6683 Å.

The scheme (in Fig. 2) for Subgroup C and for the ${}^{6}H_{11/2}$ state is considered uncertain due to the number of unaccounted-for lines in Subgroup C, which may be due to the general weak intensity of the lines of Subgroup C; as it is possible that the lines of Dy^{3+} for Subgroup C had the same intensity as lines not due to Dy³⁺. For example, it is possible that, if line no. 35 was due to the exciting source while the possible line indicated (V)above at 6616 Å actually exists, and is due to Dy³⁺, the Γ_8 and Γ_6 designations for the levels, respectively, at 5967 and 5761 $\rm cm^{-1}$ would be interchanged [since 6616 Å equals 15115 cm⁻¹, and the transition would be: $20877 \rightarrow 5761 \text{ cm}^{-1}$]. The result in this case is that the 5967 cm⁻¹ level would have no transition to the ${}^{4}F_{11/2}$, (Γ_7) 20877 cm⁻¹ level indicating Γ_6 for the 5967 cm⁻¹ level; and the 5761 cm⁻¹ level would have transitions to all the levels of the ${}^{4}F_{11/2}$ state indicating it to be Γ_{8} .

The unaccounted-for weak lines Nos. 17 and 47 may be vibrational lines. Lines Nos. 29 and 30, Nos. 9 and 10 which appear as a single one in Fig. 1 were resolved on weaker spectrograms (not shown). Line No. 13 appeared broadened by a weaker component ~ 4750 Å on stronger spectrograms than Fig. 1(A) [cf. Fig. 1(E)].



WAVE LENGTH - MILLIMICRONS (APPROXIMATE)

FIG. 1. Fluroescence of Dy^{3+} in CaF_2 , Spectrum I. The line numbers are identified in Table I. (A) Enlargment of the spectrogram of Subgroup A. (B) Enlargement of the spectrogram of Subgroup B. (C) Enlargement of the spectrogram of Subgroup C. (D) Mercury comparison spectrogram. (E) Spectrograms showing the relative positions and intensities of Subgroups A, B, and C.